Engineering Analysis I

Consider the differential equation

$$\frac{\mathrm{d}^3\phi}{\mathrm{d}x^3} - 2\frac{\mathrm{d}^2\phi}{\mathrm{d}x^2} - \frac{\mathrm{d}\phi}{\mathrm{d}x} = 0.$$

a) Transform the differential equation to a system of first-order differential equations.

b) Using matrix methods, obtain the solution to the system of differential equations.

Engineering Analysis II

a) Using the Laplace inverse

$$\mathcal{L}^{-1}\left[\mathrm{e}^{-a\sqrt{p}}; p \to t\right] = \frac{a}{\sqrt{4\pi t^3}}\mathrm{e}^{-a^2/4t}, \tag{1}$$

show that

$$\mathcal{L}^{-1}\left[\frac{\mathrm{e}^{-a\sqrt{p}}}{p}\right] = \frac{2}{\sqrt{\pi}}\int_{a/\sqrt{4t}}^{\infty} \mathrm{e}^{-x^2}\,\mathrm{d}x \equiv \operatorname{erfc}\left(\frac{a}{\sqrt{4t}}\right).$$

b) A semi-infinite bar is at a temperature T_0 above the ambient temperature at time $t = 0$. The heat conduction in the bar is governed by

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}.$$

For time $t > 0$, the end of the bar, $x = 0$, is kept at a temperature $T = 0$. Find the temperature distribution in the bar for $0 < x < \infty$, $0 < t < \infty$.

Fluid Mechanics I

The circle $|z| = a$ is a rigid surface submerged in a large body of an incompressible inviscid fluid of constant density. A source of strength m is introduced at any point outside the circle.

a) Clarify any additional assumptions you require for each part.
b) Find the resulting flow.
c) Find the resultant hydrodynamic force acting on the circular rigid surface.
d) Is the force the same if the source is replaced by a sink of the same strength?
e) Give a qualitative explanation to the answer of part c)

Fluid Mechanics II

Starting from the general form of the governing equations of incompressible flow, shown below, develop the equations governing the Falkner-Skan similarity solutions for boundary layers using the following form of the velocity:

$$U(x,y) = U(x)\, f'(\eta)$$

where

$$\eta = y / \xi(x)$$

a) Utilize your solution to obtain the governing equation and boundary conditions that represent the flow over a 30 degree wedge.

b) Outline how you may solve the problem using the results of part a)

Governing Equations:

$$\partial_i v_i = 0$$

$$\partial_o v_i + v_j \partial_j v_i = -(1/\rho)\, \partial_i P + \nu\, \partial_j \partial_j v_i + F_i$$

Thermal Sciences I

1. For a simple compressible substance whose equation of state is known, show that the isentropic exponent can be obtained from:

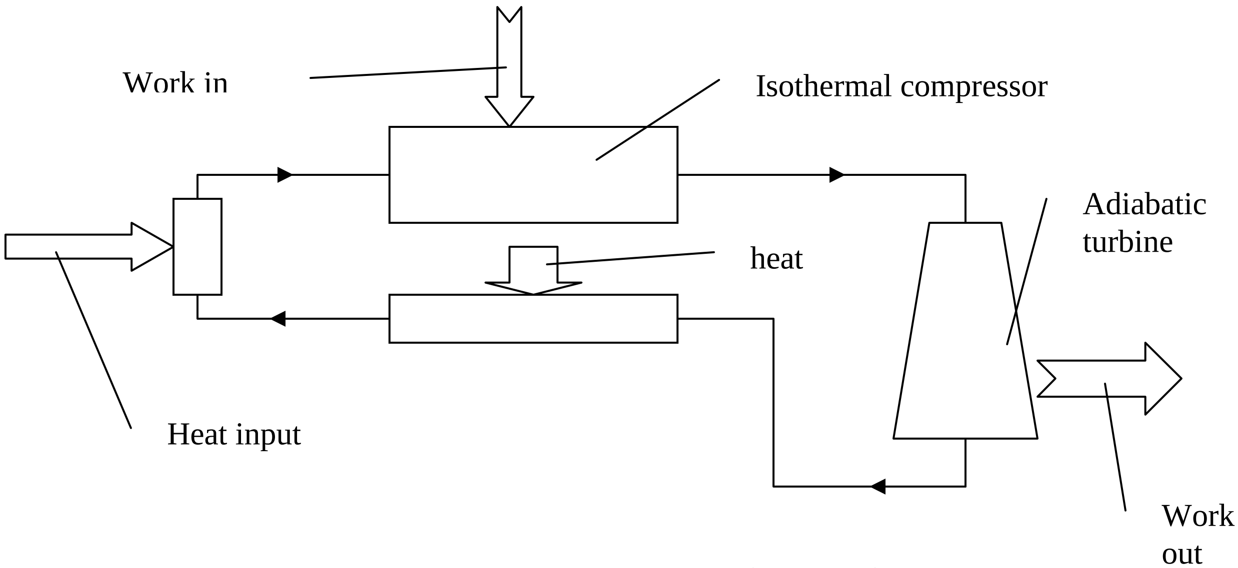
$$k = -\frac{v}{p}\left(\frac{\partial p}{\partial v}\right)_T \frac{Cp}{Cv}$$

and apply it to a substance whose equation of state is the Van der Waals equation:

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

Thermal Sciences II

An inventor has devised the machine represented by the following flow diagram:



Where the isothermal compressor is cooled by the working fluid itself, which has cooled after going through the adiabatic turbine. The working fluid specified by the inventor is refrigerant-22, but he insists it would work similarly with an ideal gas. He also insists that the compressor, because it is isothermal, will need less work to compress the fluid than the turbine yields after expansion.

Using generic values for pressures and temperatures (the inventor would not give the specific values), evaluate whether the inventor's claim can be correct. And give:

1. if the inventor's claim is correct, an example of application, for ideal gas a working fluid.
2. if it is not, a proof that his claims would be false for any particular pressure and temperatures chosen. It is not sufficient to say that a law is violated, but the specific location where it is violated, and why.

Assume all components to be ideal and frictionless