ENGINEERING ANALYSIS I

a) Given the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix},$$

determine $e^{\mathbf{A}}$.

b) Use residue theory to evaluate the integral

$$\int_{-\infty}^{\infty} \frac{1 - \cos(2x)}{x^2} dx.$$

ENGINEERING ANALYSIS II

The propagation of stress waves in a bar is governed by the wave equation

$$\frac{\partial^2 \sigma}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \sigma}{\partial t^2}, \quad c^2 = \frac{E}{\rho}.$$

A bar of length l is initially stress free. At $t = 0$ the end $x = 0$ is subjected to a constant stress of σ_0, while the end $x = l$ has the constraint $\partial\sigma/\partial x = 0$. Using the Laplace transform and a series expansion in exponential terms find the stress distribution in the bar. Sketch the distribution when $t = 3l/c$.

PhD Qualifying Exam Questions
Solid Mechanics (MMAE 530)

1. The stress tensor at a certain point of the loaded solid is given by

$$\underline{\underline{\mathbf{T}}} = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$$

(i) Find the stress vector (traction, $\underset{\sim}{t}$) on a surface with normal in the direction (0,1,1). Give the normal (σ) and the shear (τ) components of this stress vector.

(ii) Find the principal stresses and the principal axes.

2. The stress tensor in a material at rest is given by

$$\underline{\underline{\mathbf{T}}} = \frac{1}{4}\rho\omega^2 \begin{pmatrix} x_1^2 & 2x_1x_2 & 0 \\ 2x_1x_2 & x_2^2 & 0 \\ 0 & 0 & 2\left(x_1^2 + x_2^2\right) \end{pmatrix}$$

where ρ is the density (constant) and ω is constant.

Find the body force that must be acting on this body.

3. An engineer in an aerospace company is designing a rotor blade and wants to have estimates of the maximum stress and deformation in the blade. The engineer approaches you for help because of your expertise in stress analysis. The problem is simplified to the following idealized situation so that you can make a back of the envelope calculation. The blade is idealized to be a bar attached to the axis of rotation and rotation and rotating at N revolutions per second. Assume the bar is of constant cross-sectional area, A, and density ρ.

(i) Calculate the stress and displacement fields within this idealized rotor blade.

(ii) Find the maximum stress and its location.

(iii) What is the maximum displacement in the blade and its location?

(Hint: Formulate this as a 1-D problem.)

1. We are considering the large-scale ocean currents that develop under the influence of wind shear in conjunction with the Coriolis force due to the rotation of the earth. If we model the "ocean" as a layer of water of infinite depth with a flow that is induced by the shear stresses generated on the water surface by the atmospheric boundary layer, then one can show that there is an exact solution of the two-dimensional, incompressible Navier-Stokes equations corresponding to a flow with only horizontal velocity components u and v which are functions of the depth alone. We want to investigate this solution in more detail below.

In order to do so, we denote by x and y the horizontal coordinates with corresponding velocity components u and v, respectively, and z is the vertical coordinate (directed upward). Thus, for our ocean of infinite depth, we will consider $-\infty < z \leq 0$. We assume that the applied surface stress τ_0 acts in the y-direction. Assuming that horizontal pressure gradients can be neglected, the momentum equation becomes

$$\nu \frac{\mathrm{d}^2}{\mathrm{d}z^2}(u + \mathrm{i}v) - 2\mathrm{i}(u + \mathrm{i}v)\omega \sin(\phi) = 0,$$

where $\mathrm{i} = \sqrt{-1}$, ν is the kinematic viscosity, ϕ is the angle of latitude, and ω is the rotation rate of the earth, $\omega = 2\pi/d_s$, where $d_s \approx 86164.1$ s is the length of the *sidereal day*. Please note that we are using a complex representation of the velocity vector for convenience only; the flow we are interested in is *not* a potential flow. The boundary conditions on the free ocean surface are an imposed stress τ_0 in the y-direction only, so that

$$\frac{\mathrm{d}u}{\mathrm{d}z} = 0, \quad \frac{\mathrm{d}v}{\mathrm{d}z} = \frac{\tau_0}{\mu}, \quad \text{at } z = 0,$$

where $\mu = \nu\rho$ is the dynamic viscosity of the water.

a. Find the solution for the velocity components $u(z)$, $v(z)$.
Note: It can be helpful to introduce a characteristic depth D via

$$D = \pi \sqrt{\frac{\nu}{\omega \sin(\phi)}}.$$

b. Find the surface velocity of the water, and the angle between the velocity on the surface of the ocean and the wind direction.

c. What is the depth at which the water current is directed exactly opposite to the wind direction, and what is the velocity at that depth?

d. Find the magnitude and the direction of the total mass flux per unit area that is generated by the wind.

e. To get some numbers, let us take the ocean currents near Rhode Island, which has $\phi \approx 41°$. The viscosity of the water is $\mu \approx 10^{-3}$ kg/m s and its density is $\rho \approx 10^3$ kg/m^3. At a wind velocity of $V_W = 6$ m/s, there is a surface shear stress of about $\tau_0 \approx 2.64 \cdot 10^{-2}$ N/m^2. What values do you get for D and V_0? Comment.

2. Suppose there is an incompressible viscous fluid (with density ρ and kinematic viscosity ν), initially at rest and in a domain bounded by two rigid plates between $y = 0$ and $y = d$ with $-\infty < x < \infty$. At time $t = 0$, the lower plate is suddenly set into motion with a steady velocity U in its own plane, while the upper plate continues to be held stationary.

a. Derive the differential equation governing the solution of this problem, starting with the Navier-Stokes equation for an incompressible fluid.

b. What are the boundary conditions pertaining to this problem?

c. This problem has two different characteristic time scales. What are these?

d. Without solving the differential equation, determine which one of the above time scales is the one that governs the evolution of this flow. Justify your choice.

e. Find an asymptotic steady solution of the problem given by the equation you found in **a.**, with the boundary conditions from part **b.**

f. Find the unsteady solution of the problem given by the equation you found in **a.**, with the boundary conditions from part **b.**
Hints:
- It is a good idea to introduce a new velocity variable by subtracting out the solution you have found in **e.**
- A solution of the problem may be found by separation of variables.

g. After a sufficiently long time, what function gives the shape of the deviation of the unsteady solution above from the steady solution?

h. What does "after a sufficiently long time" in the question above mean?

Thermal Sciences I

Starting from the fundamental relationship between the properties of a simple compressible substance:

a. Prove that the specific heat Cp of an ideal gas ($p\,v = R\,T$) can only be a function of temperature.

b. If the substance obeys the Dieterici equation

$$p\,(v-b) = R\,T\,\exp(-a/RTv),$$

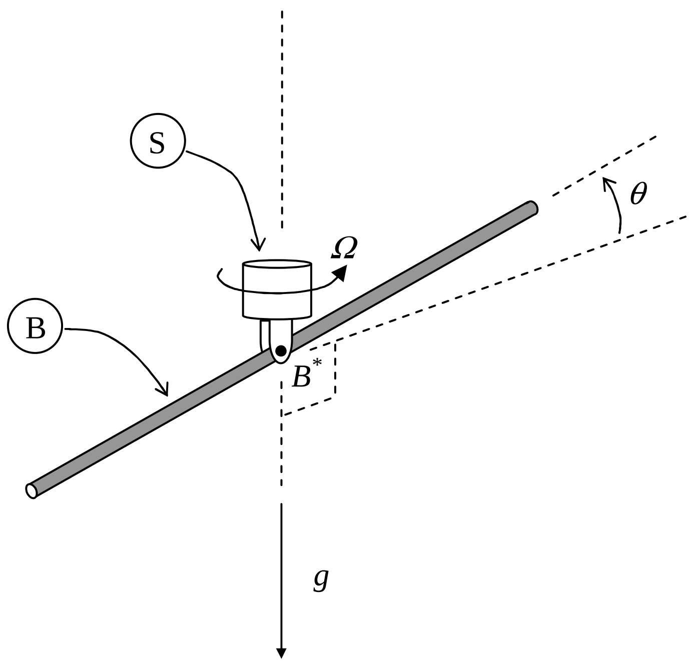
find an expression for the specific enthalpy, h, starting from Cp and the equation of state. Does Cp depend only on temperature, in this case?

Thermal Sciences II

A spacecraft carries a tank containing 1 kg of oxygen at 10 bar, 300 K. If the conditions outside are those of perfect vacuum, also at 300 K, determine the potential of that oxygen tank to produce power. Note: you may obtain a result that needs further explanation. If so, please do so, stating whether some of the usual assumptions do not apply to this case.

DYNAMICS 1

A thin rod B is connected at its mass center B^* by a frictionless pin to a shaft S. The shaft rotates with respect to an inertial frame about a vertical axis with constant rate Ω. The mass of B is m and its length is ℓ. Find the equation of motion for the rod and the reaction torque at the pin.



DYNAMICS 2

As illustrated in the figure below, a cylindrical rotor R is rotating with respect to the support structure S about axis OQ with a constant angular rate ω. The support structure S itself is rotating with respect to an inertial frame about a vertical axis with constant angular rate Ω. The mass of the rotor is m, its moment of inertia about OQ is J, and its moment of inertia about any axis passing through R^* (the center of mass of the rotor) and orthogonal to OQ is I. The distances between R^* and bearings O and Q are both ℓ. Find the value of ω that will result in zero vertical reaction force at bearing Q.

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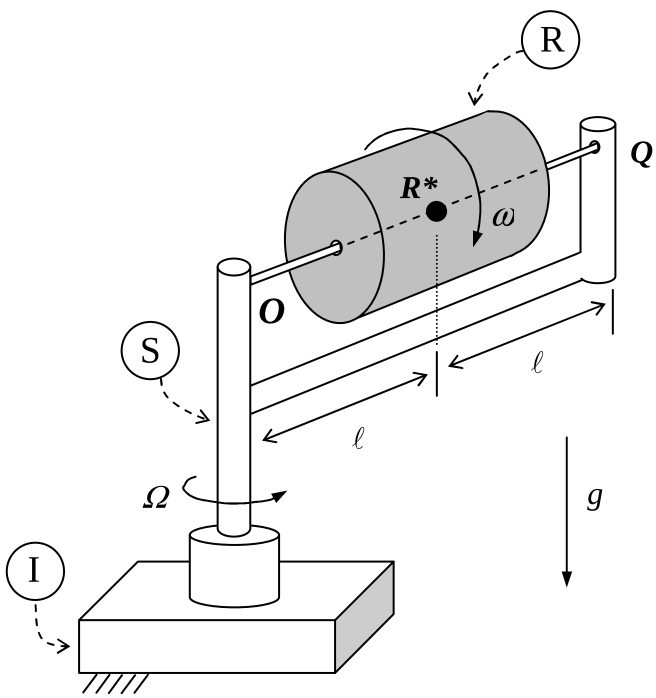

1 Design and Manufacturing I

1100-O aluminum billets 8 inches in diameter and 16 inches in length are extruded into $0.75\,\mathrm{in}$ diameter bars. Due to a change in the customer needs, the final product needed to be increased in length by 10 feet. Would it be beneficial in terms of pressure to increase the billet diameter or the billet length? Assume that the frictional work is 20% of the total work and redundant work is 30% of the frictional work. Show all necessary steps to justify your answer.

The true stress-true strain behavior of 1100-O aluminum is described by

$$\sigma = 26000\epsilon^{0.2}, \tag{1}$$

where σ is the true stress in psi and ϵ is true strain.

2 Design and Manufacturing II

a) Determine the theoretical minimum bend radius for a sheet metal $0.25\,\mathrm{in}$ thick. The sheet metal's percent reduction in area in a tensile test is 50%.

b) Determine the maximum percent reduction in area in a frictionless wire drawing process. Assume that the wire material is non-strain-hardening.