ENGINEERING ANALYSIS I

a) Obtain the general solution to the following system of differential equations using matrix operations

$$\frac{dx_1}{dt} = -x_2,$$
$$\frac{dx_2}{dt} = x_1.$$

b) Consider the mapping

$$w = \frac{1}{z},$$

where $z = x + iy$ and $w = u + iv$.

i) Determine the equation for the image in the w-plane of the line $x = 2$ in the z-plane.

ii) Determine the equation for the image in the w-plane of the line $y = 1$ in the z-plane.

ENGINEERING ANALYSIS II

Two semi-infinite bars, A: $-\infty < x < 0$ and B: $0 < x < \infty$ have diffusivities κ_A and κ_B, respectively. The conductivities are k_A and k_B. For times $t < 0$, the bars are at uniform temperatures, T_{A0} and T_{B0}. At time $t = 0$, their ends are made to contact. Using Laplace transforms, find the transient temperatures, $T_A(x, t)$ and $T_B(x, t)$.

Recall that the unsteady heat diffusion equation is

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2},$$

and that the temperatures and heat fluxes must be continuous at the interface between the two bars as follows

$$T_A = T_B, \quad \text{and} \quad k_A \frac{\partial T_A}{\partial x} = k_B \frac{\partial T_B}{\partial x} \quad \text{at} \quad x = 0.$$

A potentially useful relation:

$$\mathcal{L}^{-1}[e^{-a\sqrt{p}}] = \frac{a}{\sqrt{4\pi t^3}} e^{-a^2/4t}$$

Thermal Sciences I (525)

Consider fully developed constant-property laminar flow between parallel planes with constant heat rate per unit of length and a fully developed temperature profile. Suppose heat is transferred to the fluid on one side and out of the fluid on the other at the same rate.

Determine the Nusselt number on each side of the passage.
Sketch the temperature profile.

Suppose the fluid is an oil for which the viscosity varies greatly with temperature. Is the velocity profile affected? Is the temperature profile affected? Is the Nusselt number affected? Explain.

Problem #2

Consider fully developed laminar flow with constant properties in a circular tube. Let the surface be insulated, but let there be heat generation within the fluid at a rate S, W/m^3, that is everywhere the same.

Determine the shape of the temperature profile within the fluid.
Does the highest temperature of the fluid at any axial position occurs at the tube wall or at the tube centerline? Explain the reasons for the result.

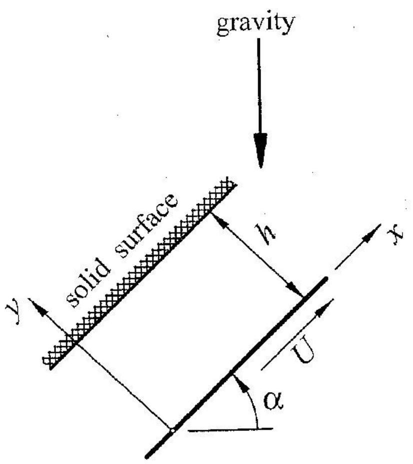
1. We are considering the flow between two infinite parallel porous plates at $y = +h$ and $y = -h$. The main flow is generated by a constant pressure gradient $-\mathrm{d}p/\mathrm{d}x$, and the porous walls are such that a uniform vertical crossflow is generated,

$$v = v_w = \text{const.}$$

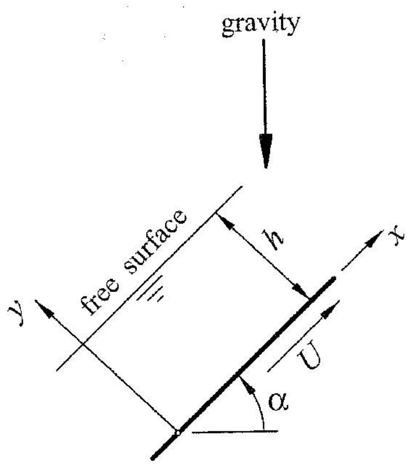
Find the velocity distribution for this flow, using the Reynolds number $Re = v_w h/\nu$ as a parameter.

- **a.** What is the maximum u-velocity $u_{\max}$ (parallel to the x-axis) in the channel?
- **b.** Give an expression for $u(y)/u_{\max}$.
- **c.** Show that in the limit of vanishing suction/blowing at the walls, your expression above reverts to the standard plane Poiseuille flow solution.
- **d.** What velocity profile do you obtain in the limit of very strong suction/blowing?
- **e.** Find the velocity profile $u(y)$ if instead of the plane channel we have steady flow at a constant velocity U (at $y = \infty$) past an infinite flat plate (at $y = 0$), with a normal velocity v_w through the wall, but $u_w = u(y = 0) = 0$. Does the sign of v_w matter? Explain your answer.

2. A moving belt is inclined at an angle to the horizontal. The lower end of this belt is immersed in a pool of liquid and the belt drags some of the liquid with it as it moves upward and out of the liquid. The liquid may be assumed to be viscous, but incompressible, and the gap width h is small compared to the distance traveled by the belt.



(a)



(b)

a. Using the configuration shown in figure (a), solve the Navier Stokes equations for the following quantities:

  i. The velocity distribution in the liquid,
  ii. the volumetric flow rate of the liquid in the x-direction,
  iii. the angle for which the volumetric flow rate is zero.

b. Repeat part **a.** for the configuration shown in figure (b).