ENGINEERING ANALYSIS I

Determine the curve $y(x)$ that minimizes

$$I = \int_0^1 (y')^2 dx, \quad y(0) = y(1) = 0,$$

subject to the constraint

$$\int_0^1 y^2 dx = 1.$$

ENGINEERING ANALYSIS II

a) State the convolution theorem for Laplace transforms.

b) Given

$$\mathcal{L}\left[\frac{1}{\sqrt{t}}\right] = \sqrt{\frac{\pi}{p}},$$

find $\mathcal{L}[\sqrt{t}]$.

c) Solve

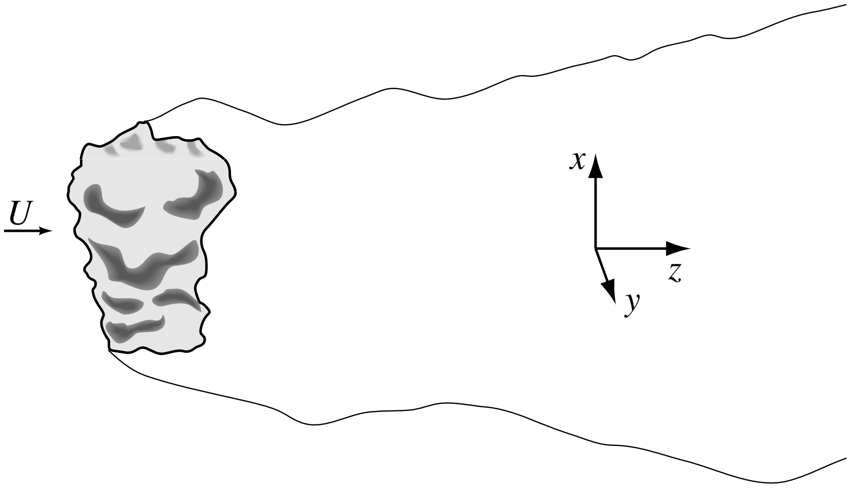
$$\int_0^x \frac{u(t)dt}{\sqrt{x^2 - t^2}} = x.$$

Qualifying Exam

Fluid Mechanics

March 12, 2004

1. A three-dimensional body of arbitrary shape produces a laminar, steady wake in an oncoming flow (see the figure below). In a cross-section z =const., the longitudinal velocity component in the wake is given by some function $u = u(x, y; z)$.



a. Derive a (simplified) equation that describes how this velocity profile evolves as we move along the z-coordinate. You can find such an equation by determining which of the terms in the full equations can be neglected because they are small. The flow is incompressible, and your equation should hold approximately in the *far wake*, i. e. for $z >> L$, with L a characteristic length of the body. Proceed from the incompressible Navier-Stokes equation,

$$\boldsymbol{u} \cdot \operatorname{grad} \boldsymbol{u} = -\operatorname{grad} p + \frac{1}{Re} \Delta \boldsymbol{u},$$

and indicate (with justification) which of the terms in this equation can be neglected in the far wake.

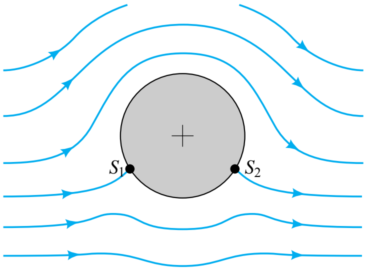
b. Based on the equation you derived in **a.**, what is the asymptotic shape of a cross-section of the wake for $z >> L$? You might want to write the simplified equation you have derived above in cylindrical coordinates. For this, you can make use of the fact that the Laplace operator in cylindrical coordinates (r, θ, z) is given by

$$\nabla^2 u = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2}.$$

In order to answer this question, you need to be able to make a statement regarding the function that describes the distribution of the longitudinal velocity component in a cross-section of the wake.

Hint: A representation of the solution using a Fourier series will be helpful.

2. We consider ideal, inviscid flow around a circular cylinder at the origin with a radius of $R = 1$, see the figure below. Such a flow can be generated by superimposing a source-sink pair (a so-called *doublet*) and a potential vortex in a parallel flow. The parallel flow can be described by a complex potential of $F_1(z) = cz$, a doublet is described by a potential of the form $F_2(z) = c/z$, and a potential vortex has a potential of $F_3(z) = ic \ln(z/d)$, where z is the complex coordinate, and c, d are real constants.



a. Write down the complex potential for this flow, assuming a free-stream velocity of U_0, and a circulation of Γ_0.

b. The lift coefficient of a body immersed in ideal two-dimensional flow is proportional to the circulation. Use dimensional analysis to determine the lift coefficient c_ℓ as a function of the relevant parameters in the case of the flow around the circular cylinder. The undetermined constant in this case has a value of unity.

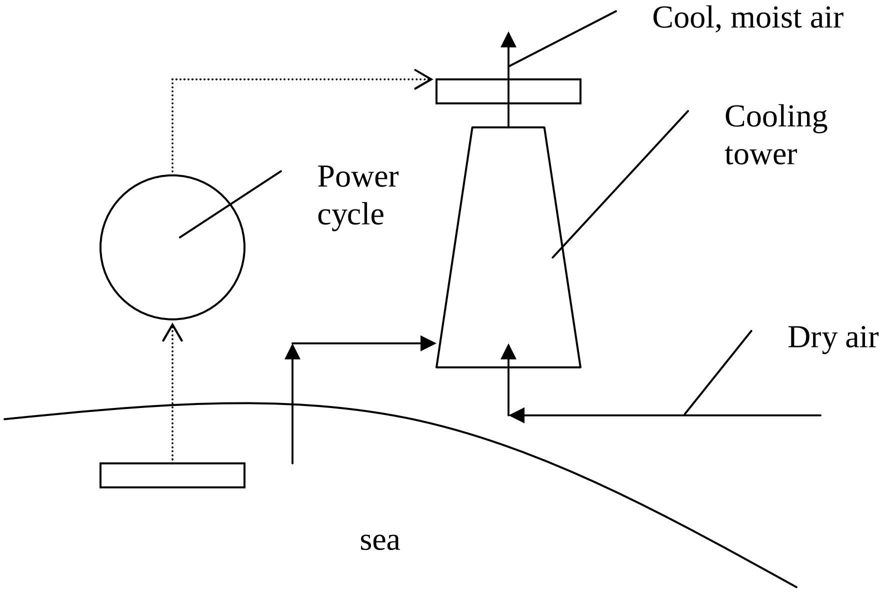
c. Find the location of the stagnation points S if the circulation is such that the above cylinder generates a lift coefficient of $c_\ell = 1$.

d. Find the lift coefficient c_ℓ when S_1 and S_2 are at the same location.

Thermal Sciences I

1. It is proposed to build a power plant on the Saharan coast, based on the following principle:
    - Take saltwater from the sea
    - Spray it into a cooling tower, which also takes in the very dry ambient air blowing from the desert.
    - The saltwater partially vaporizes, taking its heat of vaporization from the air, which cools down.
    - A thermodynamic cycle is run, with heat exchangers in the sea (ambient temperature) and the streams exiting the cooling tower (air and water assumed to exit at the same temperature).

    Using the concept of exergy, estimate the seawater flow required to generate 1 MW of power. Assume ambient temperature to be 30º C, and relative humidity 20%. Assume the saturation pressure of saltwater to be given by Raoult's law, and the initial salt concentration to be 1.5% by moles.



Thermal Sciences II

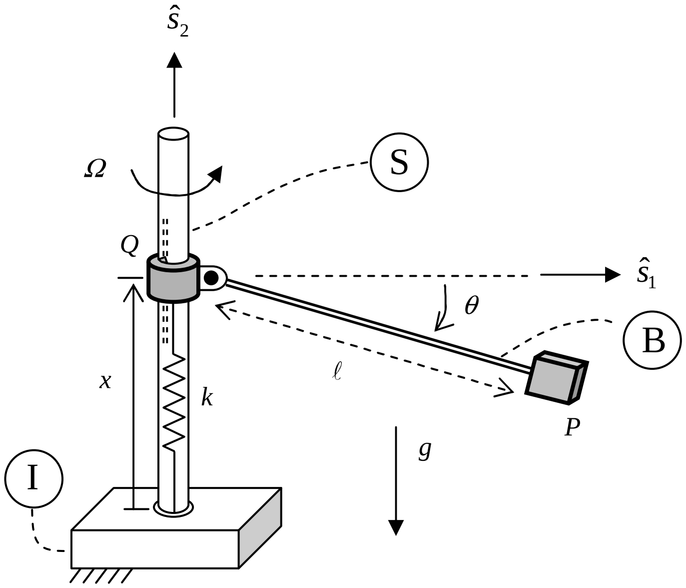
Given a real-gas equation of state f(P, T, v) = 0, how do we determine the location of the liquid-vapor saturation line? (hint, use the 2nd law of Thermodynamics over a particular cycle). Apply this result to finding the saturation line for a substance following the Van der Waals equation:

$$P = \frac{R\,T}{(v - b)} - \frac{a}{v^2}$$

DYNAMICS I

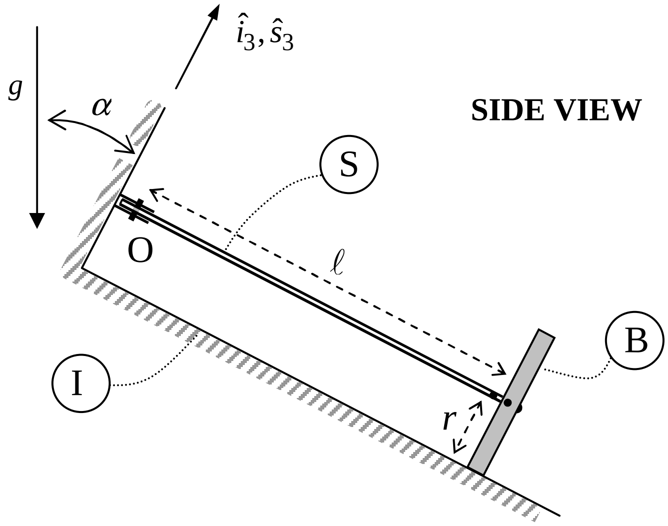
A collar ('particle' Q) of mass m_q is free to slide without friction vertically on a rod S as shown in the figure below. A spring connecting the collar to the floor (inertial frame I) has stiffness k and is unstressed when $x = x_o$. The collar is also connected to a block ('particle' P) of mass m_p by a light (i.e., massless), rigid rod B which is pinned at Q. The shaft S and rod B both spin with respect to inertial frame I at a constant angular rate Ω about the vertical axis $\hat{s}_2$. (The rod B always lies in the $\hat{s}_1$-$\hat{s}_2$ plane, which is fixed to the shaft S.)

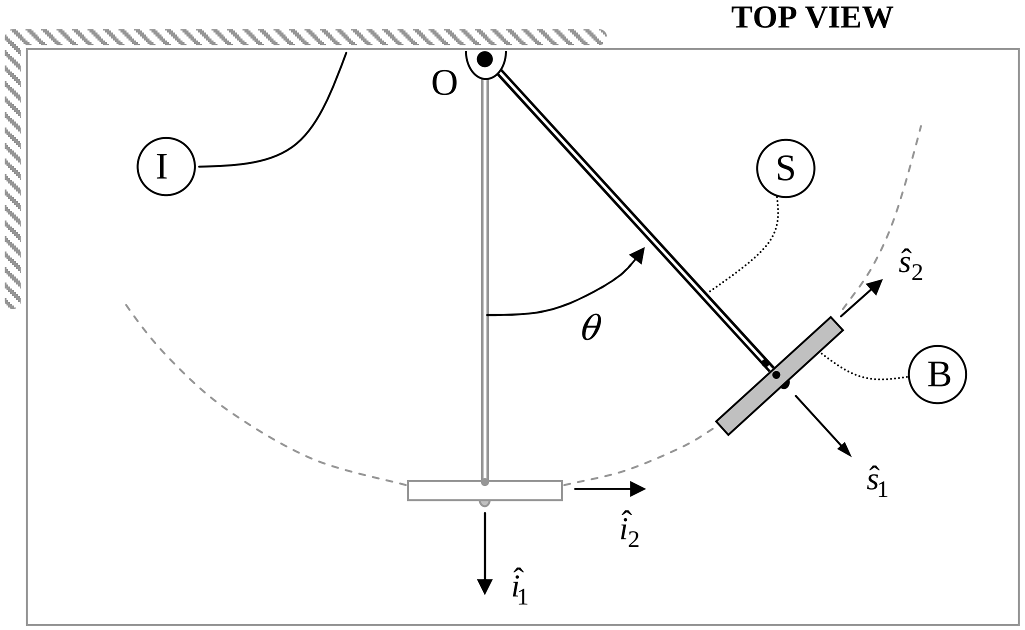
Find the equations of motion for x and θ.



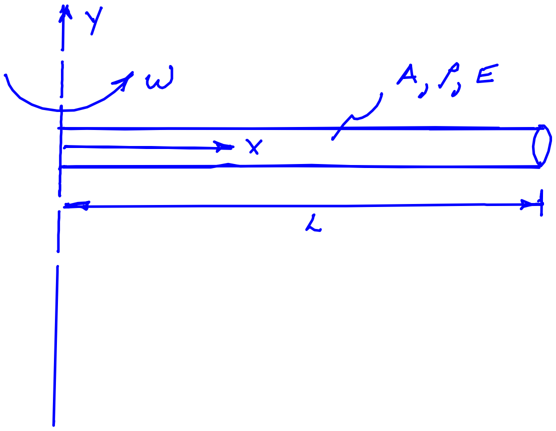
DYNAMICS II

The gyro-pendulum system shown in the figures below consists of a wheel *B* of mass *m* and radius *r* connected to a light (i.e., massless) axle *S* of length ℓ. The moments of inertia for the wheel about its own mass center are $mr^2/2$ along the direction of the axle and $mr^2/4$ for any direction perpendicular to the axle. The axle is attached to an inertial frame *I* with a pin joint *O*, and the entire mechanism is inclined with respect to horizontal by an angle α. Assuming that the wheel rolls without slipping on the inclined plane, find the equation of motion for the gyro-pendulum.





Solid Mechanics I



A linearly elastic and homogeneous bar with a circular cross-section has a constant cross-sectional area A, Young's modulus E, and mass density ρ as shown in the figure above. The bar of length L rotates at a constant angular velocity ω about the y-axis.

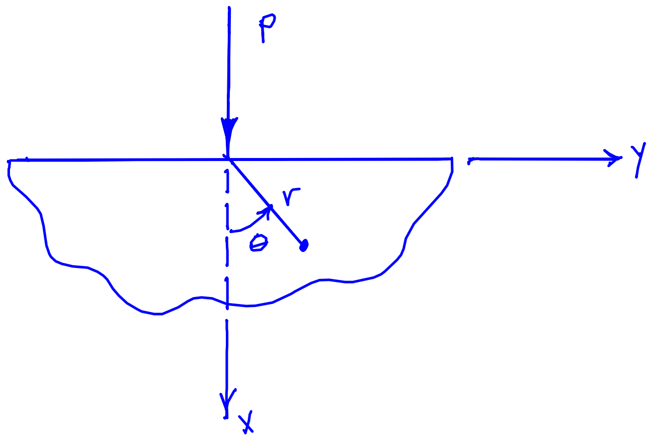
a) Derive the second order ordinary differential equation that governs the response of the bar.

b) State the boundary conditions that apply at $x=0$ and $x=L$.

c) Solve the differential equation to determine the displacement field $u(x)$ along the length of the bar.

d) Determine the stress field $\sigma(x)$ along the length of the bar.

Solid Mechanics II



Consider the problem of a half space loaded by a concentrated line load of magnitude P (force per unit length into the page). The half space is linearly elastic and isotropic.

The Airy stress function for the problem is given as

$$U = -\frac{P}{\pi} r\theta \sin\theta \qquad (1)$$

where r and θ are the polar coordinates defined in the figure.

a) Determine the stress components σ_{xx}, σ_{yy}, σ_{xy} at the point $(r, \theta) = (1, \pi/4)$.

b) Determine the stress components σ_{rr}, $\sigma_{\theta\theta}$, and $\sigma_{r\theta}$ at the point $(r, \theta) = (1, \pi/4)$.

c) Assume plane strain conditions and determine the strain components ε_{xx}, ε_{yy}, ε_{xy} at the point $(r, \theta) = (1, \pi/4)$. Take $E = 200\,\text{GPa}$ and $\nu = 0.3$.

d) Again, assuming plane strain conditions, determine the three principal stresses at the location $(r, \theta) = (1, \pi/4)$. Also, determine the three principal directions.

Hint: $\sigma_{xx} = \frac{\partial^2 U}{\partial y^2}$; $\sigma_{yy} = \frac{\partial^2 U}{\partial x^2}$; $\sigma_{xy} = -\frac{\partial^2 U}{\partial x \partial y}$

Design and Manufacturing I

a) Consider a material whose true stress versus true strain behavior can be described by the following equation:

$$\sigma = a + b\varepsilon^{1/2}$$

The numerical values of the constants a and b are given in the table below

a (psi)	*b* (psi)
20000	40000

i) Derive a relationship between stress and strain that provides the criterion for the onset of necking.

ii) Determine the ultimate tensile strength (UTS) of the material.

b) Calculate the maximum reduction in cross-sectional area in frictionless wire drawing of a rigid linearly strain hardening material whose true stress versus true strain behavior is given by:

$$\sigma = a + b\varepsilon$$

a (psi)	*b* (psi)
25000	20000

Design and Manufacturing II

a) Determine maximum reduction per pass in plane strain rolling for a sheet metal thickness of 0.25 inches if the roll diameter is 6 inches and coefficient of friction between sheet metal and rolls is 0.1.

b) State whether the following statements are true or false in reference to orthogonal machining of metals. Explain your reason for each answer.

_____ For the same shear angle, there are two rake angles that give the same cutting ratio.

_____ For the same depth of cut and rake angle, the type of cutting fluid used has no influence on the chip thickness.

_____ If the cutting speed, shear angle, and rake angle are known, the chip velocity can be calculated.

_____ The chip becomes thinner as the rake angle increases.

_____ The function of the chip breaker is to decrease the curvature of the chip.