

ENGINEERING ANALYSIS I

Problem 1

Find the solution of the following system of differential equations using matrix operations

$$\begin{aligned}\frac{dx_1}{dt} &= x_1 + 2x_2 + e^t, \\ \frac{dx_2}{dt} &= 2x_2 + t^2.\end{aligned}$$

ENGINEERING ANALYSIS I

Problem 2

A particle of mass m slides on a frictionless parabolic surface defined by $y = ax^2/2$.

- a) Obtain the Lagrangian for the motion of the particle in terms of x .
- b) Assuming small harmonic oscillations, i.e. $a^2x^2 \ll 1$, use Hamilton's principle to determine the equation of motion for the particle.
- c) For harmonic motion of the form

$$x(t) = A \cos(\omega t),$$

determine the frequency of oscillation.

PhD Qualifier Exam Spring 2009

MMAE568 Advanced Physical Metallurgy

1 a) Explain the precipitation sequence for an Al-4 wt % Cu alloy at 150°C based on the figure below and draw a schematic hardness versus aging time curve for this alloy aged at 150°C .

b) Explain why it is possible to obtain metastable precipitates before the nucleation of the stable precipitate.

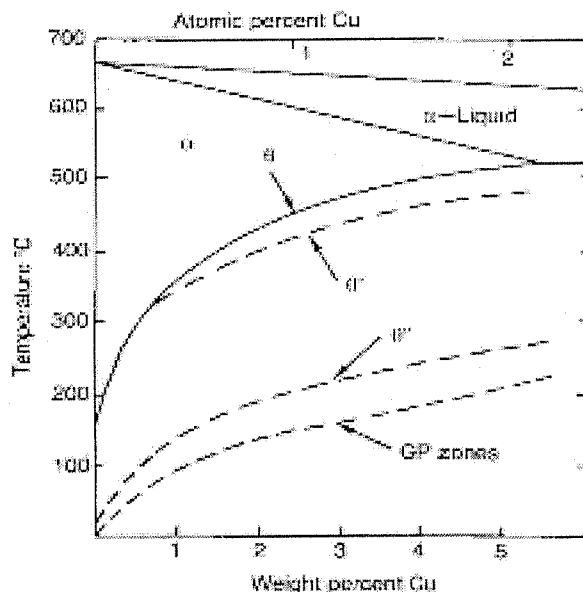


Fig. 5.25 Al-Cu phase diagram showing the metastable GP zone, θ'' and θ' subuses. (Reproduced from G. Lorimer, *Precipitation Processes in Solids*, K.C. Russell and H.I. Aaronson (Eds.), The Metallurgical Society of AIME, 1978, p. 87.)

c) Particle coarsening by volume diffusion can be expressed by:

$$\bar{r}^3 - \bar{r}_0^3 = kt$$

$$k = \frac{8\sigma(V_m)^2 C_e D}{9RT}$$

where R is the gas constant, T is absolute temperature, t is time, \bar{r}_0 is the average particle radius at time t=0 and the precipitate molar volume is V_m

What experiments would you perform to determine the rate constant k and how could you then determine the activation energy for coarsening, Q?

2. a) Draw a schematic TTT diagram. What do the curves on the diagram represent?

b) The JMA equation is: $f = 1 - \exp(-kt^n)$

How would you plot a series of data of volume fraction transformed, f , versus time, t , to obtain the rate constant k and the time exponent n ?

c) What feature of the transformation determines the time exponent, n in the JMA equation?

d) For a discontinuous (cellular) type transformation what is the effect of cold work prior to the transformation on the TTT curves?

Effect of Temperature and Stress on Creep

- A. The following data were obtained in creep tests at 816°C on a high temperature FCC metal. Assuming a single mechanism for creep is operable over the entire range of stresses determine a relationship between steady state creep rate and applied stress for this material and the value of the steady state creep rate, $\dot{\epsilon}_s$, at an applied stress of 120MPa.
- B. Determine the steady state creep rate, $\dot{\epsilon}_s$, for this material at 900°C at an applied stress of 103 MPa using the relationship below for the effect of temperature and stress on steady state creep rates (Values that are needed are $Q = 452.55$ kJ/mole, $R=8.31$ J/K, $b=0.2$ nm, $G=60$ GPa). Comment on the value obtained relative to likely experimental results and explain any differences that might be observed.
- C. Describe what metallurgical methods along with microstructures that might be used to improve the creep resistance of this alloy at elevated temperature. Indicate the likely success of each of these methods, use examples.

Stress MPa	Min Creep rate-%/hr
69	0.00008
103	0.0026
138	0.025
207	2
276	30
345	320

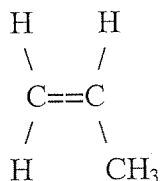
The expression for steady state creep is; $\dot{\epsilon}_s = B\sigma^n \cdot \exp\left[\frac{-Q}{RT}\right]$ for the case of variable mechanisms and different stresses.

Recall if $n=1$ Nabarro-Herring (N-H) or Coble mechanisms
 if $n=2$ viscous flow
 if $n=3 < 5 < 8$ dislocation or power law creep

Ph.D. Qualifying Exam for MMAE 470 Student(s)

Problem #1

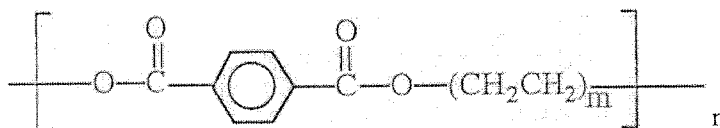
All of Problem #1 is concerned with polypropylene, often abbreviated PP. Propylene monomer is sketched below.



- A) Calculate the molecular weight of a polypropylene polymer if the degree of polymerization is 1750. Also, show the repeat unit of the polymer chain. Use atomic masses of 12 for carbon and 1 for hydrogen.
- B) Polypropylene is polymerized by a transition metal catalyzed addition polymerization reaction. What are the three basic steps of an addition polymerization reaction?
- C) Polypropylene may have three different configurational isomers – isotactic, syndiotactic and atactic. Sketch the molecular structures of isotactic and atactic polypropylene. Will either of these structures be able to crystallize? Why or why not?
- D) Crystalline PP can have an orthorhombic unit cell with dimensions of 1910 pm, 1101 pm, and 649 pm ($1 \text{ pm} = 1 \times 10^{-12} \text{ meters}$). The unit cell contains 18 repeat units per unit cell. Calculate the theoretical density of a 100% crystalline PP. Why is the answer you calculated different than the experimentally observed density of 900 kg/m^3 ?

Problem #2

- A. Consider a family of polyesters with the general structure:



How would you expect the glass transition and crystalline melting temperatures to vary with the values of m and n ? Give reasons for your answer.

- B. Sketch a plot of the modulus of an amorphous polymer as a function of temperature, labeling the different regions of viscoelastic behavior.
- C. Sketch the stress/strain curves you would expect for the following polymers:
1. Atactic Polystyrene
 2. A Butadiene Styrene Copolymer (90% butadiene)
 3. High density Polyethylene

Assume slow strain rates at room temperature. Put all 3 curves on the same plot and label the curves.

D. A sample of atactic polystyrene is separated into 5 fractions;

Fraction	Number of Moles	Molecular Weight
1	20	10,000
2	20	20,000
3	20	30,000
4	20	40,000
5	20	50,000

What is the number average molecular weight? What is the weight average molecular weight? What is the polydispersity?

MMAE 468 Introduction to Ceramics Materials

1. Two different silicon nitride plates were extensively tested to develop Weibull modulus (m) values. Tensile data for the two ceramics tested under identical conditions were as follows

<u>Ceramic</u>	<u>Mean tensile stress (σ_0)</u>	<u>Weibull modulus (m)</u>
Silicon Nitride 1	400 MPa	18
Silicon Nitride 2	450 MPa	8

The service conditions are geometrically identical to the test conditions and impose a maximum operational stress (σ) of 300 MPa.

The Weibull failure probability equation is -- $F = 1 - \exp [- (\sigma / \sigma_0)^m]$

Compare the probabilities of failure for the two different silicon nitrides at the service stress of 300 MPa.

2. You have to mill cordierite ceramic powders for a ceramic fabrication study using a ball mill. You are using the following empirical formula for milling rate

$$\text{Milling Rate} = A a^{1/2} (\rho d/r),$$

where A is the empirical factor,

“a” is the radius of the mill,

ρ is the density of the ceramic milling balls,

”d” is the starting diameter of the powders to be milled,

“r “ is the radius of the milling balls.

“A” has a value of 110 for your milling set up, your mill has a radius (a) of 9 inches, and your cordierite powder has a starting size (d) of 0.05 inches.

You can either choose alumina (density = 3.8 g/cc) milling balls with a radius of 0.25 inches or tungsten carbide (density = 14.5 g/cc) milling balls with a radius of 1.2 inches.

Which ceramic milling balls (alumina or tungsten) will give you a higher milling rate, using the milling rate equation?

ENGINEERING ANALYSIS II

Obtain the Green's function for the problem

$$x^2 u'' + 3xu' = 0, \quad u(1) = 0, \quad u(2) = 0,$$

and use it to solve the non-homogeneous differential equation

$$x^2 u'' + 3xu' = x^2, \quad u(1) = 1, \quad u(2) = 2.$$

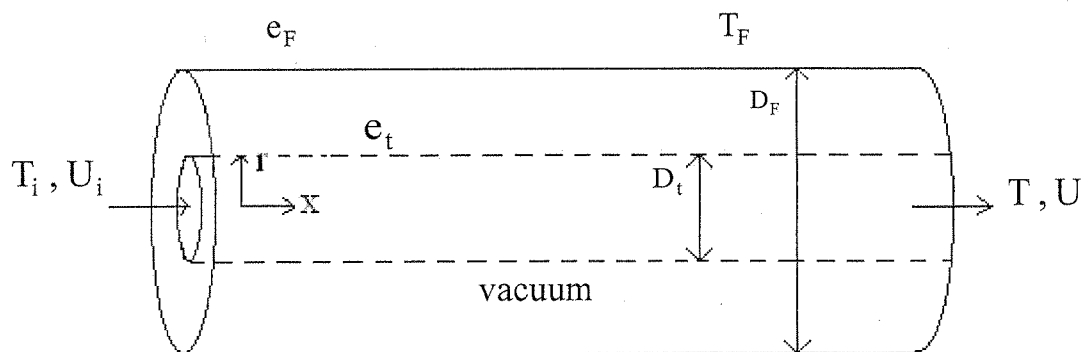
Heat Transfer I

February 2009

A tube with conductive thin wall is placed inside a cylindrical furnace as shown. The furnace internal wall is kept at high temperature of T_F . The tube diameter, D_t , is much smaller than the diameter of the furnace, D_F . The furnace is maintained in a vacuum. Let the furnace and tube emissivities be e_F and e_t , respectively. In addition, assume that the tube wall has a negligible resistance to transfer of heat.

A fluid steadily enters the tube at a uniform velocity and temperature of U_i and T_i , respectively.

- Provide the governing differential equations and the corresponding boundary conditions that if solved will yield the evolution of fluid temperature profile along the tube length. State all your assumptions. Do not solve the governing differential equations.
- Qualitatively illustrate the fluid temperature along the tube center line and the tube wall as a function of x .
- How would the formulations change if the furnace diameter is moderately larger than the tube diameter?



-Diagram not to scale

Heat Transfer II

Consider a solid sphere of radius R . Assuming spherically symmetric conditions:

- a) Derive **ONLY** the radial term of the heat diffusion equation using a variable thermal conductivity $k = C_1 + C_2r^2$ and assuming no volumetric energy generation.
- b) If two solid spheres A and B have equal radii and volumetric energy generation e''' , but A has a uniform thermal conductivity $k_A = C_1$ while B has $k_B = C_1 + C_2r^2$, sketch the ratio of the spheres' surface temperatures $T_B(R)/T_A(R)$ over a range of R as they lose heat by steady-state convection to the ambient environment. Both A and B are exposed to the same, constant values of h and T_∞ . **Explain the features of your sketch.**
- c) Now consider a solid sphere of radius R and uniform thermal conductivity k that is constructed in such a way that it exhibits volumetric energy generation e''' between r_1 and r_2 ($r_1 < r_2 < R$). Sketch the steady-state profiles of heat flux and temperature as a function of radial position. **Explain the features of your sketch.**

Design (2009S)

1. Curvature computing in a Bspline curve

For a Bspline curve defined by $(n+1)$ control points P_0, P_1, \dots, P_n , and total $2n+2$ knots with $(n+1)$ zeros and $(n+1)$ ones. What are the curvatures at the two end points of the Bspline curve?

Hint: Curvature $k(u) = \frac{|p'(u) \times p''(u)|}{|p'(u)|^3}$

Design (2009S)

2. A cubic NURBS curve defined by 9 control points is shown in the figure below. Assuming it is a non-periodic uniform curve. Assuming the knot vector is $0\ 0\ 0\ 0\ 1\ 2\ 3\ 4\ 5\ 6\ 6\ 6\ 6$. The weights for the 9 control points are $1, 2, 2, 2, 2, 2.5, 1.5, 1, 1.5$. Evaluate the curve point at $u=1.5$.

